Exploring Quantum Computing for Solving the Traveling Salesman Problem

Mia Minaya, Kahlil Dozier, Dan Rubenstein
Department of Computer Science, Columbia University

COLUMBIA UNIVERSITY

Introduction

- Quantum computing is a game-changing method of computing where it stores data not as discrete 0s and 1s but as quantum bits, or qubits, which, due to a phenomenon called superposition, may exist in more than one state at a time.
- Quantum computers' capacity to solve complex problems faster than classical computers offers promise for encryption, optimization, and scientific simulations.
- The Travelling Salesman Problem is a classic example of combinatorial optimization, in which the goal is to find the Hamiltonian cycle that goes to each city precisely once and then back to the starting point.
- IBM Qiskit's phase estimation algorithm makes use of the inverse quantum Fourier transform to convert phase information into classical values, allowing us to calculate the eigenvalues of unitary matrices.
- Our analysis replicates the 2018 study by Karthik Srinivasan, Saipriya Satyajit, Bikash K. Behera, and Prasanta K. Panigrahi on "Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience."

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

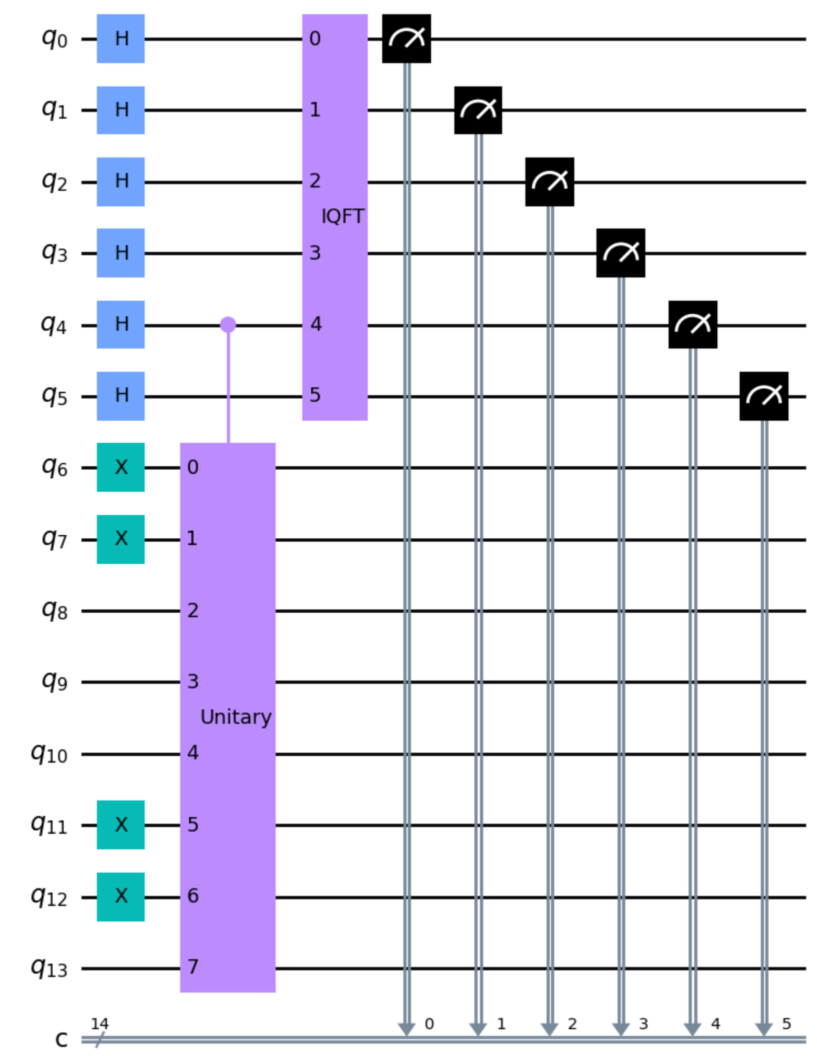
$$|\psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$$

Results

- Using the simulator, we can see the full 14-qubit circuit with all gates applied to their corresponding bits, the composite unitary gate U4 applied to the target bits, the IQFT applied to the first qubits, and the resulting measurement.
- Strings of bits show how many shots a traditional bitstring has been seen in. For every run, the Pauli x gates were applied to different qubits of the last 8 qubits.
- Bitstrings 1 and 2 match, 3 and 4 match, but 5 and 6 do not. The final output's bitstrings estimate the quantum state's phase.
- The bitstring with the most observations has the most plausible quantum circuit phase value; the related phase (distance) is more likely to be a component of the TSP's ideal path.

SL No.	Eigenstate	Theoretical	Experimental
1.	11000110	100100	000010
2.	01101100	100100	000010
3.	10001101	100000	000010
4.	01110010	100000	000010
5.	11100001	011000	111110
6.	10110100	011000	000000

{'00000000011010': 6, '00000000010110': 7, '00000000000010': 375, '00000000010010': 11, '00000000110010': 13, '00000000100010': 6, '00000000111110': 396, '00000000000110': 41, '00000000000000': 32, '00000000100110': 7, '00000000011110': 5, '00000000111010': 43, '00000000101110': 7, '00000000001010': 17, '00000000110110': 11, '00000000001110': 17, '00000000101010': 6}



Methods

- Start by representing the distances between each city as phases. Construct diagonal unitary matrices to capture these distances, ensuring that the phases are normalized within the range of 0 to 2pi.
- Transform the phases into the form cos(theta) + i * sin(theta) after normalization. Repeat this process for each unitary matrix corresponding to the cities in the graph.
- Next, implement the unitary matrices in the Qiskit quantum programming framework. Perform tensor products of the matrices.
- Apply specific unitary gates, such as the Hadamard and Pauli X gates, to targeted qubits as required.
- Use a unitary gate as the control gate, with a desired qubit being the control qubit and the rest of the qubits acting as target qubits.
- Perform the Inverse Quantum Fourier Transform on the desired qubits.
- Finally, measure the qubits that had IQFT applied and execute the code in the simulator to obtain the output.
  - On the right, we used two qubits to run on the quantum computer instead of 14 since it is too large to compute. The math is a look into how the calculations worked for 14 qubits.

$$\{P_1 \le P_2 \le P_3 \le, \ldots, \le P_n\}$$

$$\{\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_n\}$$

$$\hat{p}_i = N \cdot \frac{P_i}{P_n}$$

$$\begin{bmatrix} 1 & e^{i\frac{\pi}{20}} \\ e^{i\frac{\pi}{2}} & 1 \end{bmatrix}$$

$$\{1, 10\}$$

$$\hat{P}_1 = \frac{\pi}{2} \cdot \frac{1}{10} = \frac{\pi}{20}$$

$$U_1 = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{20}} \end{bmatrix}$$

$$\hat{P}_2 = \frac{\pi}{2} \cdot \frac{10}{10} = \frac{\pi}{2}$$

$$\left\{\frac{\pi}{20}, \frac{\pi}{2}\right\}$$

$$U_2 = \begin{bmatrix} e^{i\frac{\pi}{2}} & 0 \\ 0 & 1 \end{bmatrix}$$

Conclusion

Quantum phase estimation together with the Inverse Quantum Fourier Transform significantly outperforms classical algorithms, demonstrating the potential of quantum computing to change combinatorial optimization problems like the TSP. This provides a basis for further research into quantum technologies and for creating quantum-enhanced algorithms that potentially change a wide range of practical optimization problems. However, we were unable to run the 14-qubit circuit on a real quantum computer as there is a limit of 5-7 qubits, so there is still room to improve the number of qubits that quantum computers can handle.

Acknowledgments

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