Low-Rank Matrix Factorization for Moving Object Detection

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INTRODUCTION

- Detecting moving objects in video data is crucial for applications in surveillance, traffic monitoring, and video analytics [1, 2].
- Traditional methods often struggle with accurately isolating moving elements from static backgrounds, especially in complex environments [3].

Moving Object Detection applied in the real world

- This research explores the use of low-rank matrix factorization combined with optimization techniques to improve moving object detection.

METHODS

- The research employs both L1 and L2 norm formulations optimized using Stochastic Gradient Descent (SGD).
- The video is represented as a matrix $M \in \mathbb{R}^{m\mathrm{x}n}$
  - m: Number of pixels per frame
  - n: Number of frames
- The goal is to approximate M by decomposing it into a rank-one matrix plus noise ($M = xy^T + \epsilon$)
  - The rank-one matrix (xy^T) represents the static background.
  - The noise (ϵ) captures moving objects.

- The L2 norm minimization problem aims to find $x \in \mathbb{R}^{\mathrm{m}}$ and $y \in \mathbb{R}^{\mathrm{n}}$ that minimizes the Frobenius norm of the differences between xy^T and M:

$$J_{L2} = \min_{x,y} \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(x_i y_j - M_{ij}\right)^2$$

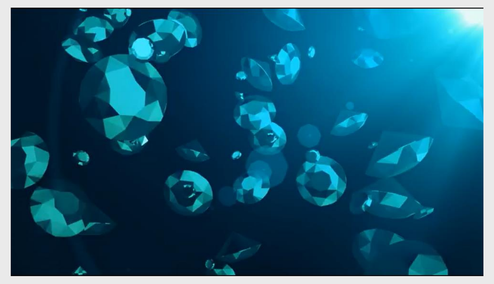
METHODS

- The L1 norm minimization problem aims to minimize the sum of absolute differences between xy^T and M:

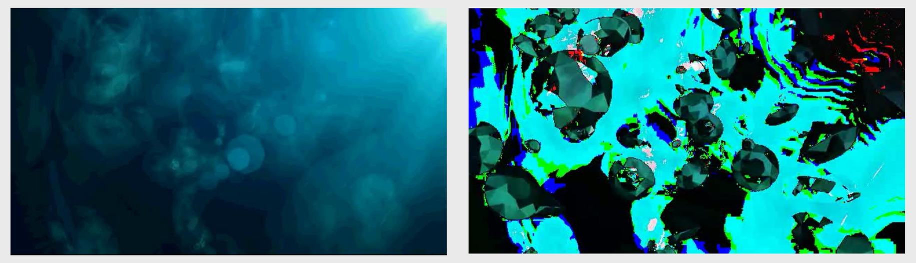
$$J_{L2} = \min_{x,y} \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \left|x_i y_j - M_{ij}\right|$$

Python Implementation:
1. **Initialize:** Start with randomly selected values for x and y
2. **Construct Matrix M**: Process video frames to construct matrix M
3. **Iterate SGD:** For each iteration, update x and y based on the chosen L1 or L2 norm.
4. **Apply Optimization:** Apply the optimization algorithm to the matrix M using the chosen norm formulation. This refines x and y to minimize the respective objective function
5. **Output Videos:** After optimization, reconstruct the video using the optimized rank-one matrix xy^T for the background, and ϵ for the moving objects (noise).

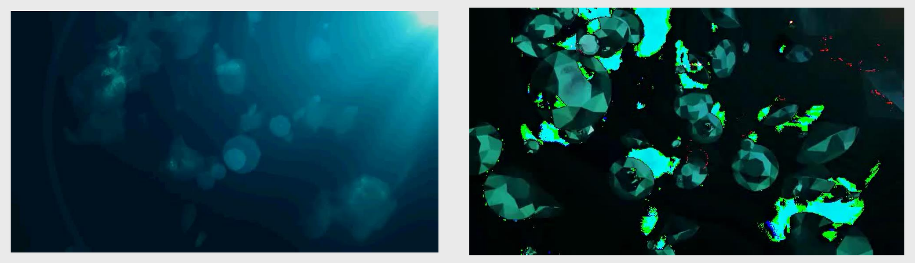
RESULTS



Example video, preprocessing (1080p, 25fps, 10sec)



Example output using the L2 norm formulation. The background (left) is smooth, but the moving objects (right) have poor separation.

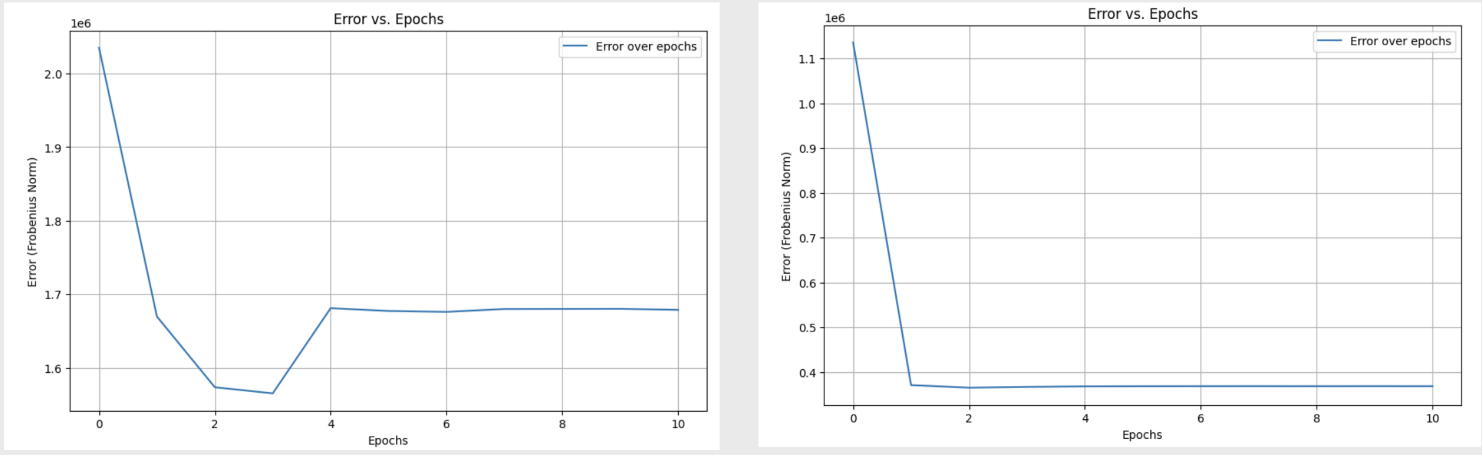


Example output using the L1 norm formulation. The background (left) and moving objects (right) are more accurately isolated.

RESULTS

- Effectiveness of Low-Rank Matrix Factorization:
  - Low-rank matrix factorization, combined with SGD, effectively detects moving objects in videos.
  - The L1 norm formulation shows significant improvements in accuracy and robustness to noise compared to the L2 norm.

- L1 Norm vs. L2 Norm:
  - L2 Norm: Provides smoother solutions but is more sensitive to outliers.
  - L1 Norm: More robust to noise and outliers, leading to better isolation of moving objects.
  - Comparative analysis shows the L1 norm improves accuracy by approximately 131% and takes significantly less time to converge



Comparative analysis of L2 (left) and L1 (right) norm formulations. The graph compares the error rates of both formulations over epochs.

BIBLIOGRAPHY/ACKNOWLEDGEMENTS

1) Bouwmans, T., et al. "Recent Advances in Background Modeling for Foreground Detection: Systematic Review and Comparative Evaluation." Computer Science Review, vol. 33, 2019, pp. 19-35.
2) Li, J., et al. "Deep Learning-Based Object Detection and Tracking in Aerial Surveillance." Sensors, vol. 20, no. 14, 2020, pp. 1-18
3) Josz, C., and Lexiao L. "Nonsmooth rank-one matrix factorization landscape." Optimization Letters, vol. 16, no. 6, 2022, pp. 1611-1631

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